Schrödinger's Reaction to epr

Erwin Schrödinger 'Discussion of probability relations between separated systems,' *Proceedings of the Cambridge Philosophical Society*, 31, 1935, 555–63; and 'Probability relations between separated systems,' *Proceedings of the Cambridge Philosophical Society*, 32, 1936, 446–52.

'When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives (or ψ-functions) have become entangled. To disentangle them we must gather further information by experiment, although we know as much as anyone could know about all that happened. Of either system, taken separately, all previous knowledge may be entirely lost, leaving us but one privelege: to restrict experiments to one only of the two systems. After reestablishing one representative by observation, the other one can be inferred simultaneously. In what follows the whole of this proceedure will be called *disentanglement*. It sinister importance is due to its being involved in every measuring process and therefore forming the basis of the quantum theory of measurement, threatening us thereby with at least a *regressus in infinitum*, since it will be noticed that the procedure itself involves measurement.

Another way of expressing the peculiar situation is: the best possible knowledge of a *whole* does not necessarily include the best possible knowledge of all its *parts*, even though they may be entirely separated and therefore virtually capable of being "best possibly known", i.e. of possessing, each of them, a representative of its own. The lack of knowledge is by no means due to the interaction being insufficiently known—at least not in the way that it could possibly be known more completely—it is due to the interaction itself.

Attention has recently been called to the obvious but disconcerting fact that even though we restrict the disentangling measurements to *one* system, the representative obtained for the *other* system is by no means independent of the particular choice of observations which we select for that purpose and which by the way are *entirely* arbitrary. It is rather discomforting that the theory should allow a system to be steered or piloted into one or the other type of state at the experimenter's mercy in spite of his having no access to it.' —1935, pp. 555–6.

In contrast with Bohr, Schrödinger gets almost everything right. He forsees

- entanglement
- non-locality, and the conflict with relativity.
- quantum holism, and the correct nature of mixtures.
- in the 1936 paper, he implies that he anticipated some of this in 1927.

Greenberger-Horne-Zeilinger Argument (in Mermin's version)
We are looking at three spin-half particles in an 8-dim Hilbert space. The operators are an extension of the 2-particle case.

1. $\mathbf{S^1_x} \otimes \mathbf{I_2} \otimes \mathbf{I_3}$
2. $\mathbf{S^1_y} \otimes \mathbf{I_2} \otimes \mathbf{I_3}$
3. $\mathbf{I_1} \otimes \mathbf{S^2_x} \otimes \mathbf{I_3}$
4. $\mathbf{I_1} \otimes \mathbf{S^2_y} \otimes \mathbf{I_3}$
5. $\mathbf{I_1} \otimes \mathbf{I_2} \otimes \mathbf{S^3_x}$
6. $\mathbf{I_1} \otimes \mathbf{I_2} \otimes \mathbf{S^3_y}$
7. $\mathbf{S^1_x} \otimes \mathbf{S^2_y} \otimes \mathbf{S^3_y}$
8. $\mathbf{S^1_y} \otimes \mathbf{S^2_x} \otimes \mathbf{S^3_y}$
9. $\mathbf{S^1_y} \otimes \mathbf{S^2_y} \otimes \mathbf{S^3_x}$
10. $\mathbf{S^1_x} \otimes \mathbf{S^2_x} \otimes \mathbf{S^3_x}$

All ten of these observables are products of commuting observables that have eigenvalues = ± 1. So the ten have eigenvalues that are ± 1. (In fact the first six are there really to allow us to prove that the last four have eigenvalues ± 1.)

Now using the three known facts about these spin operators (already proved), namely

$$\begin{aligned} \mathbf{S_x S_y} &= i\mathbf{S_z} \\ \mathbf{S_y S_x} &= -i\mathbf{S_z} \\ (\mathbf{S_x})^2 = (\mathbf{S_y})^2 = (\mathbf{S_z})^2 &= I \end{aligned} \tag{6}$$

it can be proven that

$$(\mathbf{S^1_x S^2_y S^3_y}) \cdot (\mathbf{S^1_y S^2_x S^3_y}) \cdot (\mathbf{S^1_y S^2_y S^3_x}) = -(\mathbf{S^1_x S^2_x S^3_x}) \tag{7}$$

In (7) the first three operators commute pairwise, and we have already noted have eigenvalues ± 1. Thus so does the RHS.

The proof is:

$$\begin{aligned} LHS &= (\mathbf{S^1_x} \otimes \mathbf{S^2_y} \otimes \mathbf{S^3_y}) \cdot (\mathbf{S^1_y S^1_y} \otimes \mathbf{S^2_x S^2_y} \otimes \mathbf{S^3_y S^3_x}) \\ &= (\mathbf{S^1_x} \otimes \mathbf{S^2_y} \otimes \mathbf{S^3_y}) \cdot (\mathbf{I_1} \otimes i\mathbf{S^2_z} \otimes -i\mathbf{S^3_z}) \\ &= \mathbf{S^1_x I_1} \otimes i\mathbf{S^2_y S^2_z} \otimes -i\mathbf{S^3_y S^3_z} \\ &= \mathbf{S^1_x} \otimes ii\mathbf{S^2_x} \otimes -ii\mathbf{S^3_x} \\ &= -(\mathbf{S^1_x} \otimes \mathbf{S^2_x} \otimes \mathbf{S^3_x}) \end{aligned} \tag{8}$$

Consider some state Φ which is a simultaneous eigenstate of the three commuting operators on the LHS, with eigenvalue 1. Then Φ must be an eigenstate of $\mathbf{S^1_x S^2_x S^3_x}$, but the eigenvalue must be -1. Take three particles that are space-like separated from one another in the state Φ and pick any two of the particles for measurement along the y-axis—say, 2 and 3. From the result we can predict with certainty the value of $\mathbf{S^1_x}$ on particle 1, because the product of all three must equal 1. Or we could predict with certainty the value of $\mathbf{S^1_x}$ by measuring particles 2 and 3 on the x-axis, and here the product of all three must equal -1. And finally, we could measure 2 along the x-axis and 3 along the y-axis to predict with certainty 1 along the y-axis ($\mathbf{S^1_x}$). So $\mathbf{S^1_x}$ or $\mathbf{S^1_y}$ can be predicted with certainty by making measurements on space-like separated systems. So if Locality and the Criterion of Physical Reality hold then both eigenvalues that *could* be obtained must be pre-existent properties. (Note we could also do this choosing to measure 1 and 3 to predict 2.)

From this we can conclude that all three particles must have definite values (before they are measured) and, by Locality, the values can't change as a result of the distant measurements. These 6 numbers—-$m^1_x, m^1_y, m^2_x, m^2_y, m^3_x, m^3_y$—must be such that

$$\begin{aligned} m^1_x m^2_y m^3_y &= 1 \\ m^1_y m^2_x m^3_y &= 1 \\ m^1_y m^2_y m^3_x &= 1 \\ m^1_x m^2_x m^3_x &= -1 \end{aligned} \tag{9}$$

Note each number m^i_α occurs twice on the LHS's and is ± 1. So the 12 numbers of the collective LHS's multiplies to 1, whereas the collective RHS's multiply to -1. (Take some m^i_α—because it occurs twice, if it is -1 the product is 1 and if it is 1 the product is 1. Same for the other 5. So the product of all twelve must be 1.) So there can be no pre-assigned numbers m^i_α that will satisfy (8). QED.

This shows that Separability and Locality cannot both be true. One or both must be false. In QM both, of course, are false. The opposite of separability is *entanglement*. So this demonstrates the necessity of entangled, nonlocal states. We don't need to do experiments to confirm this—it is a mathematical necessity.

See N.D. Mermin 'Simple unified form of the major no-hidden-variable theorems' *Physics Review Letters*, 65, pp. 3373–6.

N.D. Mermin 'Hidden variables and the two theorems of John Bell,' *Reviews of Modern Physics*, 65, pp. 803–815.

Geometry of Entanglement

Entanglement, which is the opposite of separability, comes in degress, and measures of entanglement have been proposed.

In 2002 Bertleman, Narnhofer, and Thirring (in *Physical Review*, A. 66, 032319) proposed a measure based on the Hilbert-Schmidt distance. This reveals the geometry of entangled states and their relation to the separable states in the total space of states.

1. The most entangled states are pure states, and mixing decreases the entanglement.
   - This is what we see in the EPR singlet state: it is entangled and pure. The marginal states are maximally mixed.
   - But note that not violating Bell's inequality is *not* the same as not entangled. Some entangled states satisfy Bell's inequality.
   - There is a neighborhood of an entangled state in which all the states are entangled.
   - There is a neighborhood of an separable state in which all the states are separable.
2. The state space decomposes into equivalence classes of states with the same entanglement.
   - All pure separable states are in the same equivalence class.
   - Obviously all maximally entangled states are also in the same equivalence class.
3. In the state space there is a plane around the most separable state. From it emerge valleys of separable states leading to the pure separable states on the boundary. In their neighborhood are entangled states which slope up from the valley bottoms. These "hills" lead to the maximally entangled states on the boundary.
4. Nevertheless we shouldn't let our intuitions become too cozy. The state space K of two spin half particles is inscribed in a 15-dimensional "ball". (If $\dim(\mathscr{H}) = n$ then $\dim(K) = n^2 - 1$—as we saw before.)
   - This should make it clear just how many more, and what different kinds of, states two particles together have over the two particles separately. It is 15 vs 3 + 3.
   - For two spin-1 particles it would be $2^6 - 1 = 63$.

QM via Information Theory

Clifton, Bub and Halvorson have recently (2003) proven a theorem which uses information theory to characterise QM—effectively deriving QM from just three information-theoretic axioms.

1. No superluminal transfer of information via measurement.
   - By performing local measrements in a region no one can tell that some other measurements have been made in a space-like separated region. The statistics will look the same to the one at a distance.
   - Every element of the C^*-algebra associated with one system commutes with every element of the C^*-algebra of the other.
2. No broadcasting the information contained in an unknown state.
   - in broadcasting a ready state ρ of B, and the state to be broadcast σ of A, are transformed into a new state ω of A + B where the marginal states of A and B are both σ.
   - For pure states this reduces to no cloning.
     - ⋆ Suppose B is a system in ready state ρ and A is a system in state σ then cloning means that B can be transformed into σ.
   - In QM neither broadcasting nor cloning are possible.
   - Implies that the C^*-algebra of a quantum system is non-commutative.
3. No unconditionally secure bit-committment.
   - a protocol in which a bit of information is committed by Alice to another party, Bob, such that Alice cannot change her committment and Bob cannot determine Alice's committment until provided with more information by Alice. (In cryptography this could be a public key.)
   - the idea is to use a density operator and Alice witholds information about which decomposition she intends.
   - but it has been shown that if it is concealing against Bob then it is not binding on Alice.
   - Alice can cheat by—after her initial committment is made—steering the remote system into any state she chooses.

The idea is to take all the theories that can be characterised by C^*-algebras—which is essentially every physical theory that has ever been known, classical or quantum—and to add these axioms to them, to pin the algebra down to just the one that characterises QM.

In fact for theories expressed as C^*-algebras, 1 and 2 jointly entail 3. So 1 and 2 together imply QM. (Proven by Halvorson, forthcoming.) And 1 and 2 imply the existence of nonlocal entangled states.

Classical mechanics is characterised by 1 and the denial of 2. So classical mechanical systems have commutative C^*-algebras, and for two systems they commute pairwise. So cloning and broadcasting are possible. However there is no unconditionally secure bit committment. So 3 holds.

Consider possible mechanics:

- If for the separated systems the respective C^*-algebras *did not pairwise commute* then superluminal messaging would be possible.
- the possibility envisaged by Schrödinger of rapidly decaying entanglement on separation is *not describable* by a C^*-algebra. And of course it is now ruled out by experiment.

Correlations and Mixed States

Here is a proof that the ignorance interpretation of mixtures cannot be maintained.

Suppose we were to have a pure state Ψ of $A+B$ and a decomposition into A in the mixed state $W_A = aP_{\psi_1} + (1-a)P_{\psi_2}$ and B in the mixed state $W_B = bP_{\varphi_1} + (1-b)P_{\varphi_2}$ then if the ignorance interpretation were correct A would be in either P_{ψ_1} or P_{ψ_2} and B would be in either P_{φ_1} or P_{φ_2}. There are four possible combinations here,

$$\psi_1 \otimes \phi_1,\ \psi_1 \otimes \phi_2,\ \psi_2 \otimes \phi_1,\ \psi_2 \otimes \phi_2 \tag{10}$$

where the probabilities (coefficients) are multiplied together (because the possibilities are conjoined). The four are

$$ab,\ a(1-b),\ (1-a)b,\ (1-a)(1-b).$$

These four probabilities will sum to 1, because

$$ab + a - ab + b - ab + 1 - b - a + ab = 1$$

and thus the composite system will be a mixed state not a pure state, contrary to our assumption that the composite state was pure.Thus if a pure composite system decomposes into component systems whose states are mixed it is impossible that these mixed states be pure states viewed under conditions of ignorance. In short, the ignorance interpretation of mixed staes must be false.[1]

(Ignorance interpretation advocated by von Neumann, and Reichenbach. Failure of ignorance interpretation argued by Margenau, Hooker, van Fraassen, Beltrametti and Cassinelli, etc. Now seems completely accepted.)

Classical Conditionalisation Rule: $\mathcal{P}(A|B) = \frac{p(A\&B)}{p(B)} = \frac{p(A\cap B)}{p(B)}$

Lüder's Rule: $\mathcal{P}(L_A|L_B) = \frac{Tr(P_B D P_B P_A)}{Tr(D P_B)}$

Cross-terms resulting from interaction

$$\begin{aligned}(\psi_1+\psi_2)\otimes(\phi_1+\phi_2) &= [(\psi_1+\psi_2)\otimes\phi_1] + [(\psi_1+\psi_2)\otimes\phi_2] \\ &= (\psi_1\otimes\phi_1) + (\psi_2\otimes\phi_1) + (\psi_1\otimes\phi_2) + (\psi_2\otimes\phi_2) \\ &\neq (\psi_1\otimes\phi_1) + (\psi_2\otimes\phi_2)\end{aligned}$$

[1]The idea of this proof is found in a number of places. The version that occurs here is taken from R.I.G. Hughes' *The Structure and Interpretation of Quantum Mechanics* Harvard University Press, 1989, p. 150.